# Atmospheric Modelling Functions

Reference:V:\Scuba-2\Documentation\Loading and Noise Analysis\blackbody_functions3.mcd(R)

Read transmission data from Excel file $data :=$

V:\..\halfmm.xls

$$trans(\nu, pwv) := \begin{cases} (tr \leftarrow data^{\langle 1 \rangle}) & \text{if } pwv = 0.5 \\ (tr \leftarrow data^{\langle 2 \rangle}) & \text{if } pwv = 1 \\ (tr \leftarrow data^{\langle 3 \rangle}) & \text{if } pwv = 5 \\ (tr \leftarrow data^{\langle 1 \rangle}) & \text{otherwise} \\ vs \leftarrow cspline(data^{\langle 0 \rangle} \cdot GHz, tr) \\ interp(vs, data^{\langle 0 \rangle} \cdot GHz, tr, \nu) \end{cases}$$

$$Z(x) := \begin{cases} \ln(x) & \text{if } x > 0 \\ (-40) & \text{otherwise} \end{cases}$$

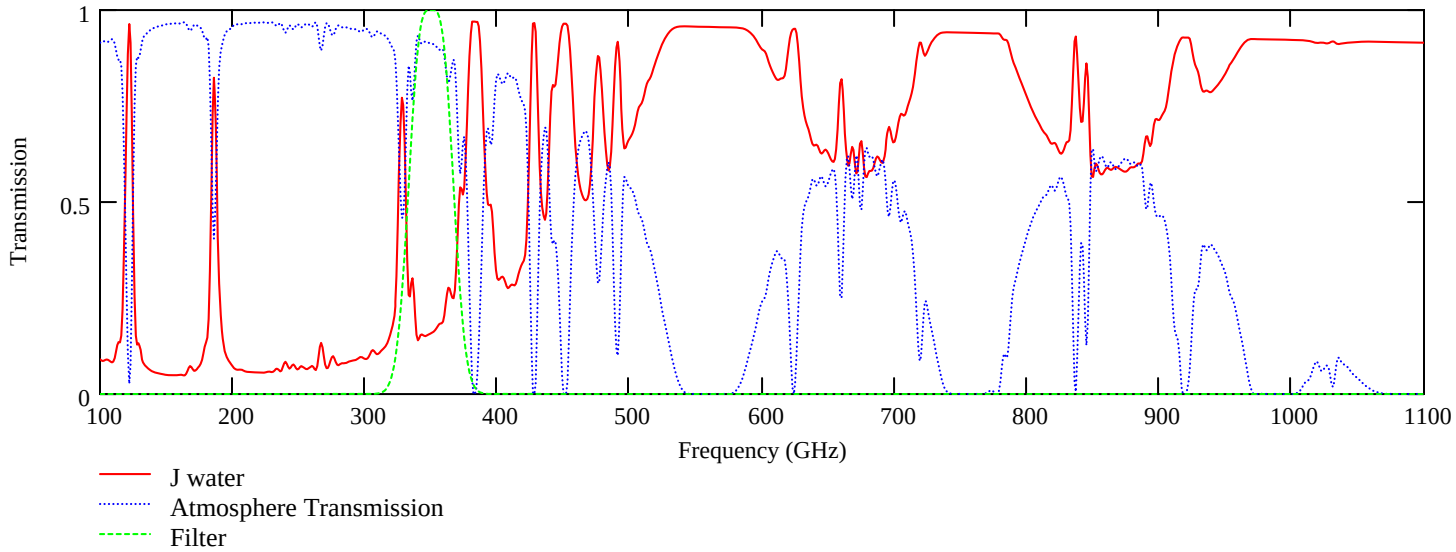
This function takes care of the regions where the atmospheric transmission arrays a,b, or d are zero.

$\nu := 100GHz, 101GHz .. 1100GHz$

$$f(\nu, d\nu, \nu_c, N) := \frac{\exp\left[\left(\left(\nu - \nu_c - \frac{d\nu}{2}\right)\right)^2 \cdot \frac{-2}{d\nu^2}\right] + \exp\left[\left(\left(\nu - \nu_c + \frac{d\nu}{2}\right)\right)^2 \cdot \frac{-2}{d\nu^2}\right]}{2\exp\left(\frac{-1}{2}\right) \cdot N}$$

$$J_{water}(A, pwv, T, \nu) := \overrightarrow{\left[J(\nu, 300K) \cdot (1 - \exp(A \cdot Z(trans(\nu, pwv))))\right]}$$

$$Atrans(A, pwv, \nu) := \overrightarrow{\exp(A \cdot Z(trans(\nu, pwv)))}$$



$$B(\nu, Temp) := \overrightarrow{\left(\frac{2\cdot h\cdot \nu^3}{c^2}\cdot\frac{1}{\exp\left(\frac{h\cdot\nu}{k\cdot Temp}\right)-1}\right)}\cdot\frac{1}{Ster}$$

$$B_{water}(A, pwv, T, \nu_c, \Delta\nu, \nu) := f\left(\nu, \frac{\Delta\nu}{2}, \nu_c, 1.\right)\cdot\left(\frac{c}{\nu}\right)^2\cdot\overrightarrow{B(\nu, T)}\cdot(1 - Atrans(A, pwv, \nu))$$

$$P_{water}(A, pwv, T, \nu_c, \Delta\nu, tr) := tr\cdot\int_{100GHz}^{1100GHz} B_{water}(A, pwv, T, \nu_c, \Delta\nu, \nu)\, d\nu$$

$$Trans(A, pwv, \nu_c, \Delta\nu) := \frac{\int_{100GHz}^{1100GHz} f\left(\nu, \frac{\Delta\nu}{2}, \nu_c, 1\right)\cdot Atrans(A, pwv, \nu)\, d\nu}{\int_{100GHz}^{1100GHz} f\left(\nu, \frac{\Delta\nu}{2}, \nu_c, 1\right) d\nu}$$

$Trans(3, 5, 345\cdot GHz, 35\cdot GHz) = 0.048$ $P_{water}(5, 5, 280\cdot K, 349\cdot GHz, 35\cdot GHz, 0.32\cdot 0.143) = 12.32\,pW$

$$Photnsq(A, pwv, T, \nu_c, \Delta\nu, trans, modes) := 2\cdot h\cdot trans\cdot\int_{100GHz}^{1100GHz} B_{water}(A, pwv, T, \nu_c, \Delta\nu, \nu)\cdot\nu\, d\nu$$

$$Photnsq(1, 0.5, 269\cdot K, 666\cdot GHz, 60\cdot GHz, 1, 0)^{\frac{1}{2}} = 4.2674\,\frac{10^{-16}\,watt}{\sqrt{Hz}}$$

$$Photnsq(1, 0.5, 269\cdot K, 666\cdot GHz, 60\cdot GHz, 0.313\cdot 0.572, 0.572) = 3.26\,10^{-32}\cdot\frac{watt^2}{Hz}$$

$TOL := 10^{-11}$ Tolerance must be this low for correct integration

The integral limits could be reduced to reduce calculation time.

$TOL := 10^{-6}$ These Integrals can use higher tolerance

Shot noise term

$$\left(P_{water}(1, 0.5, 269\cdot K, 666\cdot GHz, 60\cdot GHz, 1)\cdot 2\cdot h\cdot 666\cdot GHz\right)^{\frac{1}{2}} = 4.271\,\frac{10^{-16}\cdot watt}{\sqrt{Hz}}$$

$$\mathrm{coh}(\mu) \equiv \frac{0.33 + \exp\left(-\mu^{2.5}\right)}{1.33 + \mu^{2.5}}$$

Coherence function for number of modes in the beam for photon noise calculation, $\mu$ is the number of modes between source and detector

$\mathrm{coh}(0.5) = 0.775$

$u := 0, 0.1 .. 1$

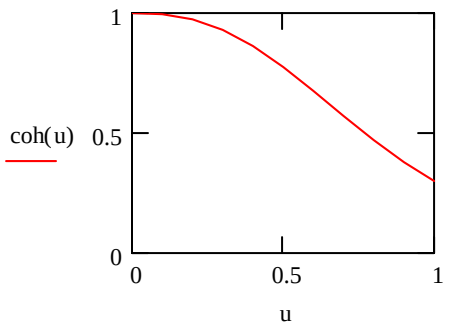
$\mathrm{coh}(1000)^{\frac{1}{2}} = 1.022 \times 10^{-4}$

$$\frac{2 \cdot h \cdot 666 \cdot GHz \cdot 60 \cdot GHz}{P_{water}(1, 0.5, 269 \cdot K, 666 \cdot GHz, 60 \cdot GHz, 1)} = 0.256$$

$\mathrm{coh}(1) = 0.3$

$1.807 \cdot 1.431 = 2.586$

$\mathrm{coh}(0) = 1$




$$4.272 \cdot \left(1 + \frac{1}{0.256}\right)^{\frac{1}{2}} = 9.463$$

$h = 6.626\,10^{-34} \cdot joule \cdot sec$

$$Ration(A, pwv, T, \nu_c, \Delta\nu, trans, modes) := \frac{2 \cdot h \cdot \int_{100GHz}^{1100GHz} B_{water}(A, pwv, T, \nu_c, \Delta\nu, \nu) \cdot \nu \, d\nu}{trans \cdot coh(modes) \cdot \int_{100GHz}^{1100GHz} B_{water}(A, pwv, T, \nu_c, \Delta\nu, \nu)^2 \, d\nu}$$

$Ration(1, 0.5, 269 \cdot K, 666 \cdot GHz, 60 \cdot GHz, 1, 0) = 0.44$