Integration during Sampling

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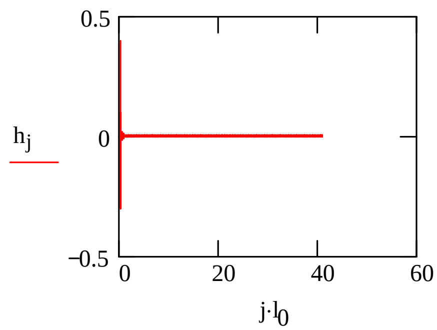
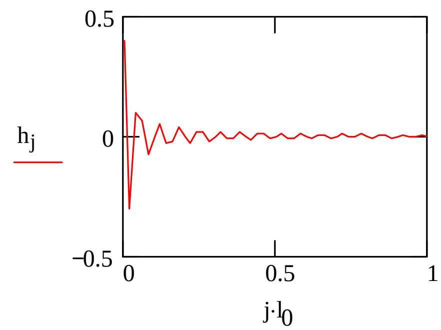
While sampling an interferogram it can be an advantage not to stop the variation in optical path difference during the sampling process. If the data acquisition integrates between the sample points a "sample" is actually an integral over the optical path variation between two samples points. Using a term from sample and hold theory the Aperture time is long compared to the sample interval.

The effect of a large aperture time is a suppression of high frequencies in the restored spectra. As an example consider a square frequency spectra from $f_N/2$-b to $f_N/2$ (measured in inverse cm). Using Mathcad with f_N =50 and b=10 the interferogram needs to be sampled with optical path step of 0.02 cm to avoid aliasing. Thus

$i := 1..2047 \qquad j := 0..2047 \qquad l_0 := 0.02 \qquad f_N := 50 \qquad b := 10$

$$h_i := \frac{2 \cdot b}{f_N} \frac{\sin(\pi\, b \cdot i \cdot l_0)}{\pi \cdot b \cdot i \cdot l_0} \cdot \cos\left[2\pi \frac{(f_N - b)}{2} \cdot i \cdot l_0\right] \qquad h_0 := 2 \cdot \frac{b}{f_N}$$



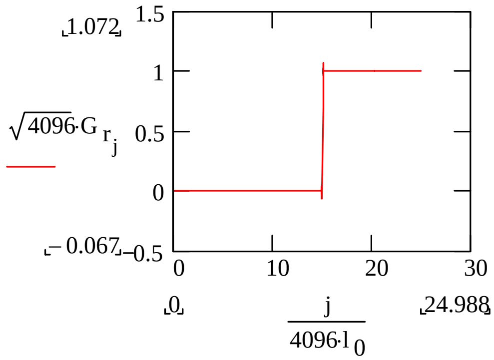


The interferogram is symmetric around zero so we really have 2*2048-1 points. This implies that the Discrete Fourier Transform pair is real and symmetric with the corresponding period. To use the Mathcad built in fft function we need to add the missing points. We will also insert an extra point to get exactly 2048 points. To guard against numerical imperfections we take the real part of the fft transform (the imaginary part is very small). The fft routine only requires the input function to be real and defined in exactly 2^n points.

$h_{i+2048} := h_{2048-i} \qquad h_{2048} := h_{2047}$

$$G := \mathrm{fft}(h) \qquad G_r := \frac{G + \overline{G}}{2}$$

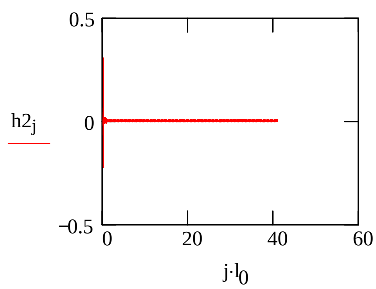
We can now compute the spectra from the sampled interferogram, which should be 0 up to $f_N/2$-b (50/2-10=15cm^{-1}) and then 1 up to $f_N/2$ (50/2=25 cm^{-1}). As can be seen in the figure below this works out fine with just a small overshoot caused by the (unrealistic) discontinuity in the bandpass. Note that the Mathcad normalization includes a square root of 4096, which we need to remove to get the right amplitude.

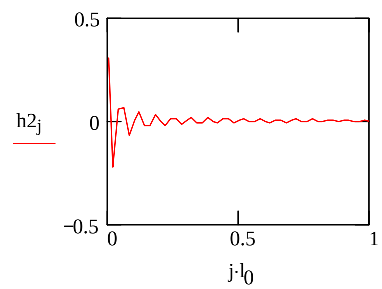


If we integrate over the sampling interval instead of sampling a discrete point the result will change. Assume the integral is over the distance between two Nyquist sample points in the optical path delay space. To make it simple we consider each sample to be centered on the interval of 0.02cm between the samples.

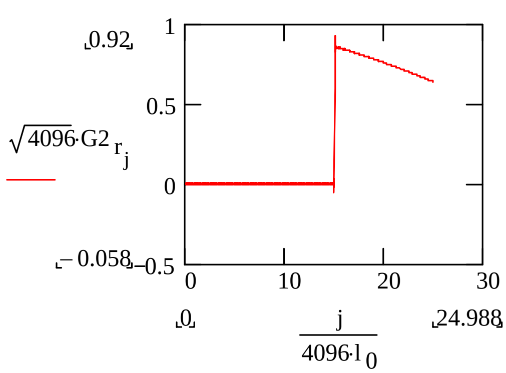
$$h2_i := \frac{2}{l_0}\cdot\int_{(i-0.5)\cdot l_0}^{(i+0.5)\cdot l_0} \frac{b}{f_N}\cdot\frac{\sin(\pi\cdot b\cdot l)}{\pi\cdot b\cdot l}\cdot\cos\left[2\cdot\pi\cdot\frac{(f_N-b)}{2}\cdot l\right] dl$$

$$h2_0 := \frac{2}{l_0}\cdot\int_{-0.5\cdot l_0}^{0.5\cdot l_0} \frac{b}{f_N}\cdot\frac{\sin(\pi\cdot b\cdot l)}{\pi\cdot b\cdot l}\cdot\cos\left[2\cdot\pi\cdot\frac{(f_N-b)}{2}\cdot l\right] dl$$



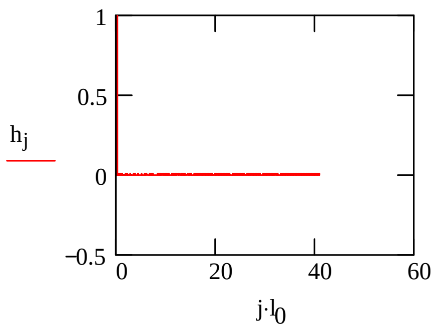


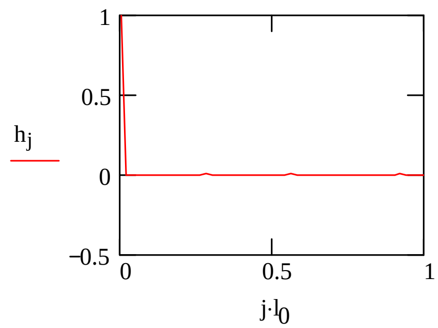
If we now compute the spectra as above we will see that the high frequency end have been reduced. This is the effect of smoothing the interferogram by integrating over the Nyquist interval.



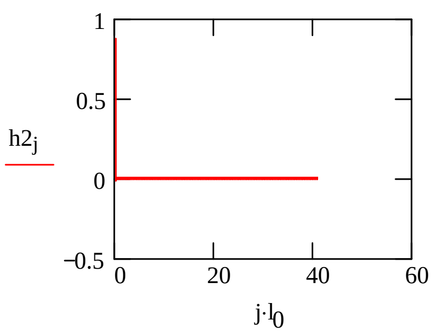
To see the effect at all frequencies we repeat the calculations with a bandpass equal to 1 up to the Nyquist frequency.

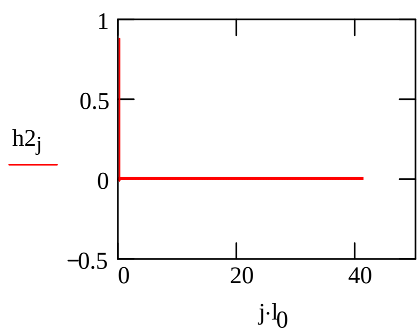
$$h_i := \frac{\sin\left(\pi \cdot f_N \cdot i \cdot l_0\right)}{\pi \cdot f_N \cdot i \cdot l_0} \qquad\qquad h_0 := 1$$





$$h2_i := \frac{1}{l_0} \cdot \int_{(i-0.5) \cdot l_0}^{(i+0.5) \cdot l_0} \frac{\sin\left(\pi \cdot f_N \cdot l\right)}{\pi \cdot f_N \cdot l}\, dl \qquad\qquad h2_0 := \frac{1}{l_0} \cdot \int_{-0.5 \cdot l_0}^{0.5 \cdot l_0} \frac{\sin\left(\pi \cdot f_N \cdot l\right)}{\pi \cdot f_N \cdot l}\, dl$$





The two resulting spectra are shown below (left – sampled, right – integration between the sample points).