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1.0	1/11/06	All	Initial CDR release
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Contents

Introduction

In the case of an ideal interferometer, an interferogram is a real and even function of the optical path difference (OPD) between the interfering radiation beams. Therefore, only one side of the interferograms need be recorded and the spectral distribution of the radiation is given by a cosine Fourier cosine transform; a one-sided transform.

In practice, due to optical, electronic or sampling effects, interferometers may depart from this ideal and have a path difference that varies with optical frequency. The interferograms are no longer symmetric and a simple one-sided transform will no longer accurately reproduce the spectrum. In particular, when an interferogram is sampled at discrete points which do not include zero path difference (ZPD), the sampling then makes a symmetric interferogram appear asymmetric.

The algorithm described in this document first corrects the asymmetry of an interferogram so that a single-sided transform may be used. This correction requires the calculation of the instrumental phase from the interferogram itself. Although double-sided transforms are required to calculate the phase, since the phase can be assumed to be slowly varying, high resolution is not needed and only a small extension of the interferogram beyond zero is required.

1. Discrete Interferograms

In practice, only a finite range of an interferogram will be sampled discretely, given by x ranging from $-L_1$ to $+L_2$, where x is the OPD. When an interferogram is sampled evenly in OPD and the interval between two adjacent sampling points is Δ, a discrete interferogram can be expressed as follows:

$$f(i\Delta),\ i = -(M_1-1), -(M_1-2), \ldots, 0, 1, \ldots, (M_2-1), M_2.$$

When $M_1=0$, this interferogram is said to be single-sided. When $M_1=M_2$, this interferogram is said to be double-sided. In general, to maximize the spectral resolution attainable with a fixed translation stage, $M_1 \ll M_2$.

As discussed above, if the interferogram is not sampled precisely at ZPD it will not appear to be symmetric around the expected ZPD position.

The short double-sided interferogram,

$$f(i\Delta),\ i = -(M_1-1), -(M_1-2), \ldots, 0, 1, \ldots, (M_1-1), M_1$$

is used to extract the instrumental phase, which leads to spectral errors if it is not corrected. In practice, the instrumental phase is determined over the whole spectrum by fitting a low order polynomial weighted by the corresponding amplitudes values of the low resolution spectrum.

2. Procedure of Phase Correction

In this algorithm, we use the following discrete Fourier transform (DFT). The forward DFT of a 1-d complex array X of size *n* computes an array Y, where:

$$Y_k = \sum_{j=0}^{n-1} X_j e^{-2\pi jk\sqrt{-1}/n}$$

The reverse DFT becomes:

$$Y_k = \sum_{j=0}^{n-1} X_j e^{2\pi jk\sqrt{-1}/n}$$

For those who like to think in terms of positive and negative frequencies, this means that the positive frequencies are stored in the first half of the output and the negative frequencies are stored in reverse order in the second half, i.e., the frequency $-k/n$ is the same as the frequency $(n-k)/n$.

A real forward DFT is used to calculate the Fourier transform coefficients of the double-sided part of an interferogram, $g(j\Delta) = f((j-M_1+1)\Delta)$, $j = 0, 1, \ldots, (2M_1-1)$. The phase and amplitude for each frequency, $\omega=k\pi/(M_1\Delta)$ ($k = 0, 1, \ldots, (2M_1-1)$), can be determined from these Fourier transform coefficients. Phase values are uniquely determined only in the range between π and $-\pi$. Care must be taken to account for those cases where phase discontinuities exist (i.e. where the phase rolls by 2π).

In general, we define the phase and amplitude in the above method as $\varphi(k)$ and $W(k)$, where $W(k)\geq 0$ and $k = 0, 1, \ldots, M_1$. The values of $\varphi(k)$ and $W(k)$, where $k = -(M_1-1), -(M_1-2), \ldots, -1$, can be obtained according to the symmetry of the real forward DFT.

The following procedure shows how to obtain the real phase from $\varphi(k)$:

1. Remove the phase offset, $\varphi(k) = \varphi(k) - k\pi/M_1$, $k = 0, 1, \ldots, M_1$.
2. When $-\pi\leq\varphi(k)<0$, $\varphi(k) = \varphi(k) + \pi$; When $-2\pi<\varphi(k)<-\pi$, $\varphi(k) = \varphi(k) + 2\pi$.
3. Define an array, $U(k)$, $k = 0, 1, \ldots, (2M_1-1)$ and $U(0)=0$.
4. When $|\varphi(k)-\varphi(k-1)| > 5\pi/6$, if $\varphi(k)>\varphi(k-1)$, $U(k) = U(k-1)-1$; if $\varphi(k)<\varphi(k-1)$, $U(k) = U(k-1)+1$. Otherwise, $U(k) = U(k-1)$.
5. The real phase will be $\varphi(k) = \varphi(k) + \pi \times U(k)$.
6. In our computation, $W(0)$ and $W(M_1)$ are set to zero. These new values of $\varphi(k)$ and $W(k)$ ($W(k)$ will be the weight factor) are used to do a polynomial weighted least-square fitting to obtain the phase errors over the whole spectrum. Note: only the points whose W is greater than a predefined value (e.g., $0.1 \times W_{max}$) are taken into account in this fitting. From this fitting polynomial, get the new value of $\varphi(k)$, $k = 0, 1, \ldots, M_1$.
7. If multiple bands exist in the spectrum, the piece-wise fitting method will be used. Firstly, each individual band is fitted by a polynomial as described in the previous step. Now, assume Point M to be the middle point of the gap between two neighbouring bands: lower wavenumber band, b1, and higher wavenumber band, b2. Extrapolate band b1 upward to Point M and get the phase of M, p1. Extrapolate band b2 downward to Point M and get the phase of M, p2. If p1 > p2

and N = [$\frac{p1-p2}{\pi}$ + 0.5] ([x] is the integer part of number x), relatively up-shift the phases of band b2 by Nπ. If p1 < p2 and N = [$\frac{p2-p1}{\pi}$ + 0.5]. Then, all modified phases will used in phase-fitting as described in the previous step.

8. Construct the butterflied phase, $\varphi(k)$, k = -(M_1-1), -(M_1-2), …, 0, 1, …, M_1, where $\varphi(-k)=-\varphi(k)$.
9. The phase correction function will be $PCF(x) = FT^{-1}(e^{-i\varphi(\omega=\frac{k\pi}{M_1\Delta})})$, k = -(M_1-1), -(M_1-2), …, 0, 1, …, M_1.

3. Numerical Tests

A real interferogram is expressed as

$$I(x) = \int_0^\infty d\omega\, f(\omega)\cos(\omega x + \phi(\omega)),$$

where, $\phi(\omega)$ is the phase. In our numerical tests, we assume that the phase includes a constant term, a linear term and a quadratic term, i.e.:

$$\phi(\omega)=c_0+c_1\omega+c_2\omega^2.$$

In a discrete scheme,

$$\begin{aligned}
&\phi(\omega=2\pi k/(2M_1\Delta)=\pi k/(M_1\Delta))\\
&= c_0+c_1\omega+c_2\omega^2\\
&= c_0+c_1\,(\pi k/(M_1\Delta))+c_2(\pi k/(M_1\Delta))^2\\
&= C_0+C_1(\pi k/M_1)+C_2(\pi k/MB_1)^2,
\end{aligned}$$

where, $C_0 = c_0$, $C_1 = c_1/\Delta$ and $C_2= c_2/\Delta^2$.

In this section, we study three cases with different C_0 , C_1 and C_2. The size of the double-sided part is 600 (i.e., M_1=300) and the size of the single-sided part is 5000 (i.e., M_2=5000) and the size of the phase correction function is 200. Phase values at frequencies whose corresponding intensity values fall below a certain threshold (in this case set to be 10% of the maxima over the whole spectrum) are not included in the phase-fitting routine.

Case 1: $C_0 = 0.4\pi = 1.2566$, $C_1 = 6.0$, $C_2 = 0.2$

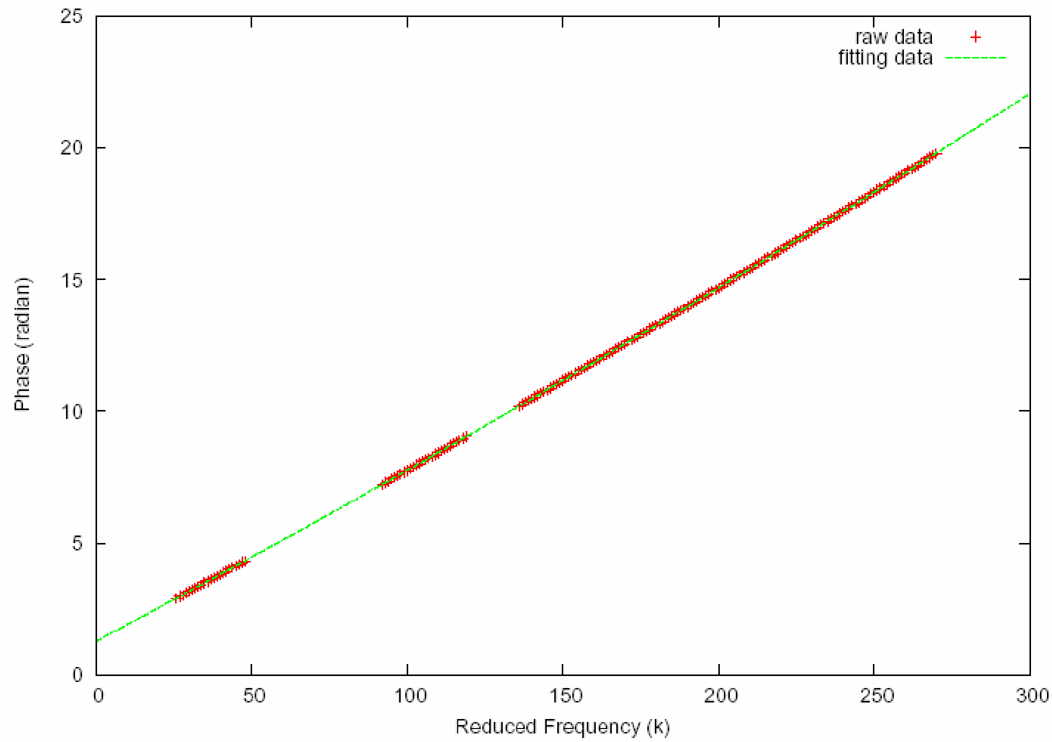

Figure 1. Result of the phase fitting procedure. The red crosses represent the raw phase data and the green line is the fit phase. The fitting parameters are C_0 =1.257, C_1 = 5.999 and C_2= 0.200.

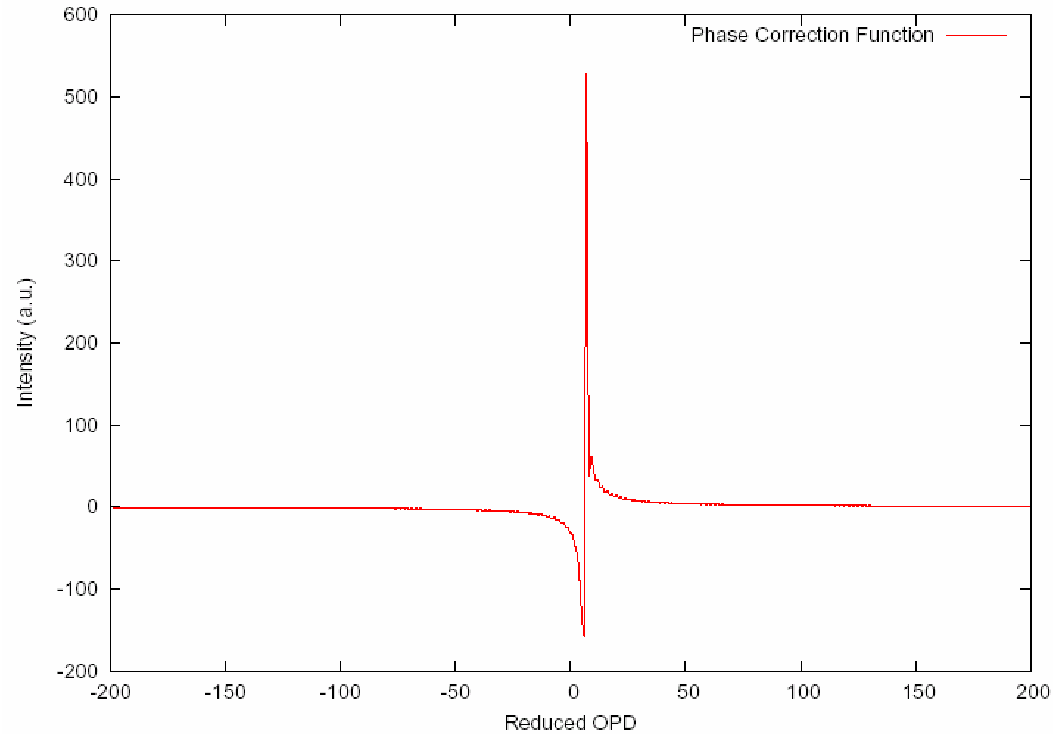

Figure 2. The Phase Correction Function with C_0 =1.257, C_1 = 5.999 and C_2= 0.200.

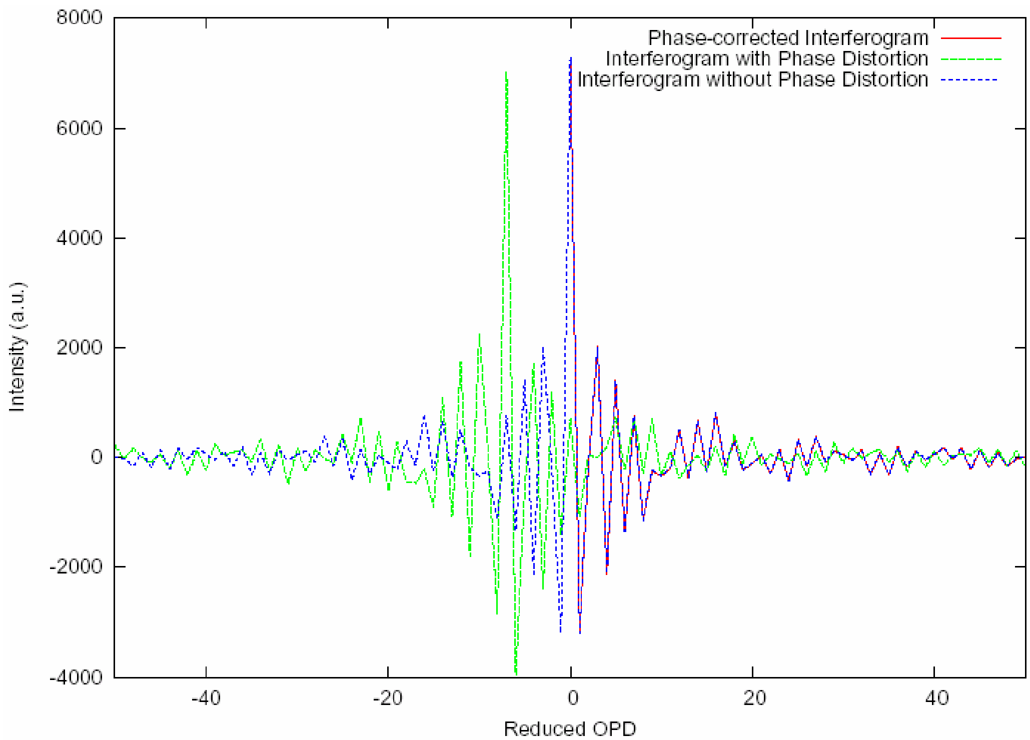

Figure 3. Interferogrms around the ZPD. The red line represents the phase-corrected interferogram (no value in the left side of the ZPD), the green dash line represents the interferogram with phase errors (C_0=0.4π, C_1=6.0, C_2=0.2), and the blue dot line represents the interferogram without phase errors (C_0=0, C_1=0, C_2=0). In the right side of the ZPD, the red line and the blue line overlays each other.

Case 2: $C_0 = 0.4\pi = 1.2566$, $C_1 = 3.0$, $C_2 = 3.0$

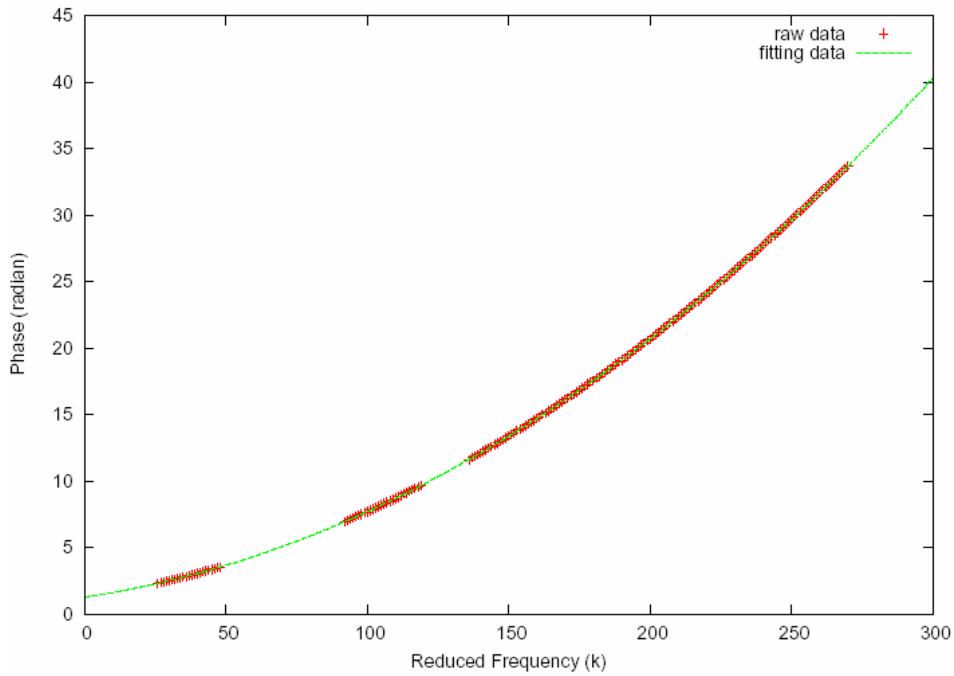

Figure 4. Result of the phase fitting procedure. The red crosses represent the raw phase data and the green line is the fit phase. The fitting parameters are C_0=1.257, C_1=2.999 and C_2=3.000.

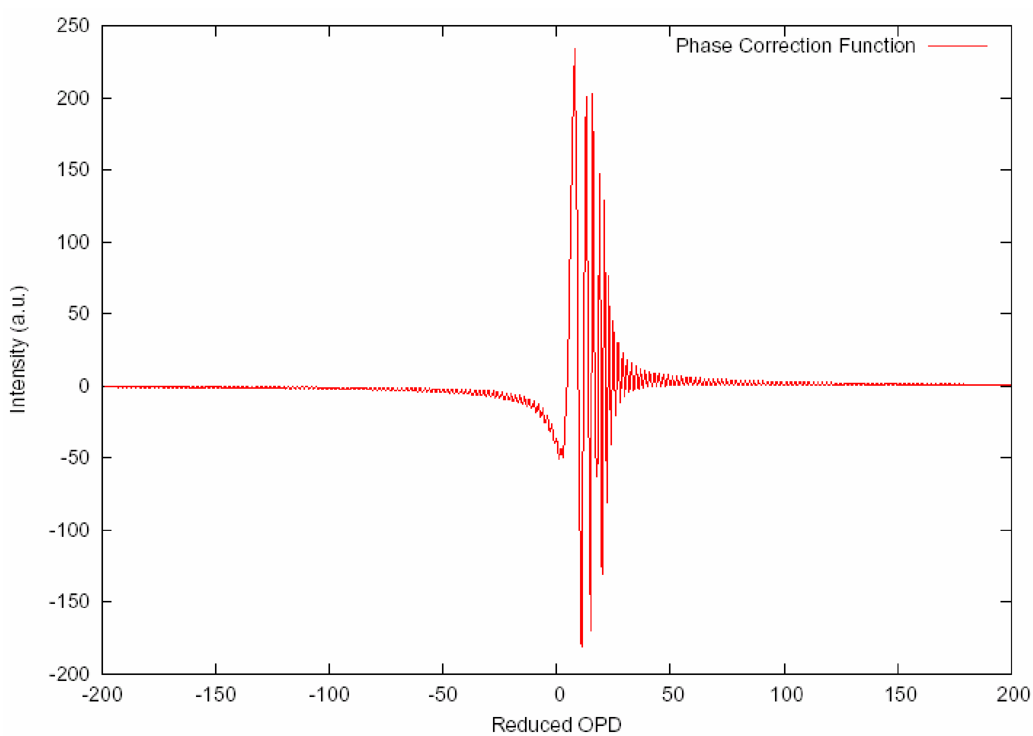

Figure 5. The Phase Correction Function with C_0=1.257, C_1=2.999 and C_2=3.000.

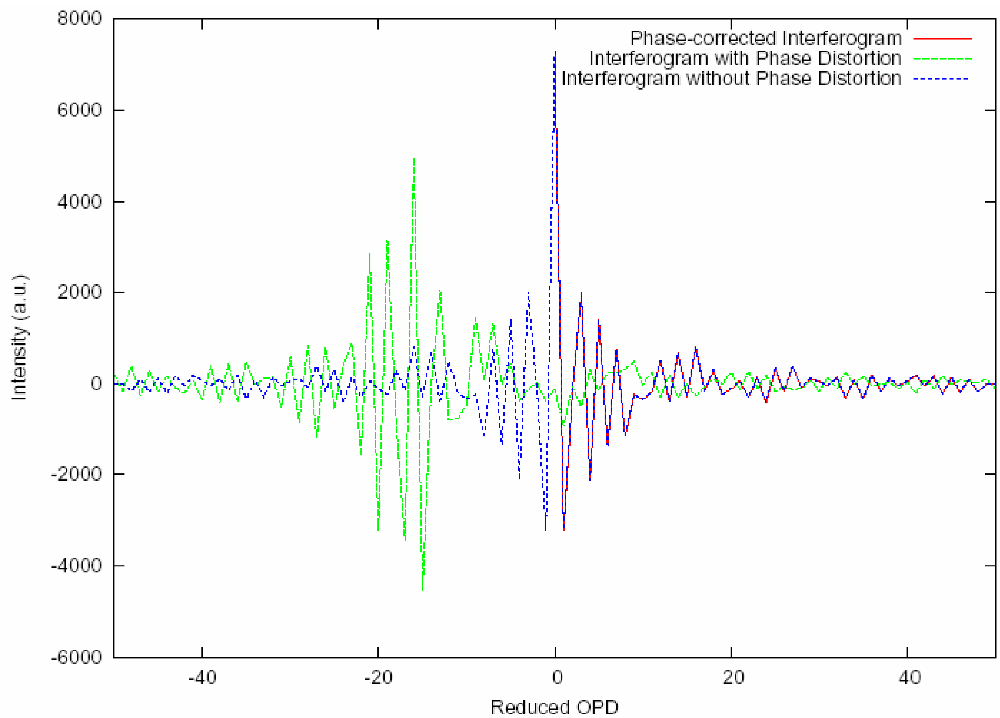

Figure 6. Interferograms around the ZPD location. The red line represents the phase-corrected interferogram (no value in the left side of the ZPD), the green dash line represents the interferogram with phase errors (C_0=0.4π, C_1=3.0, C_2=3.0), and the blue dot line represents the interferogram without phase errors (C_0=0, C_1=0, C_2=0). In the right side of the ZPD, the red line and the blue line overlays each other.

Case 3: $C_0 = 0.4\pi = 1.2566$, $C_1 = -6.0$, $C_2 = 4.0$

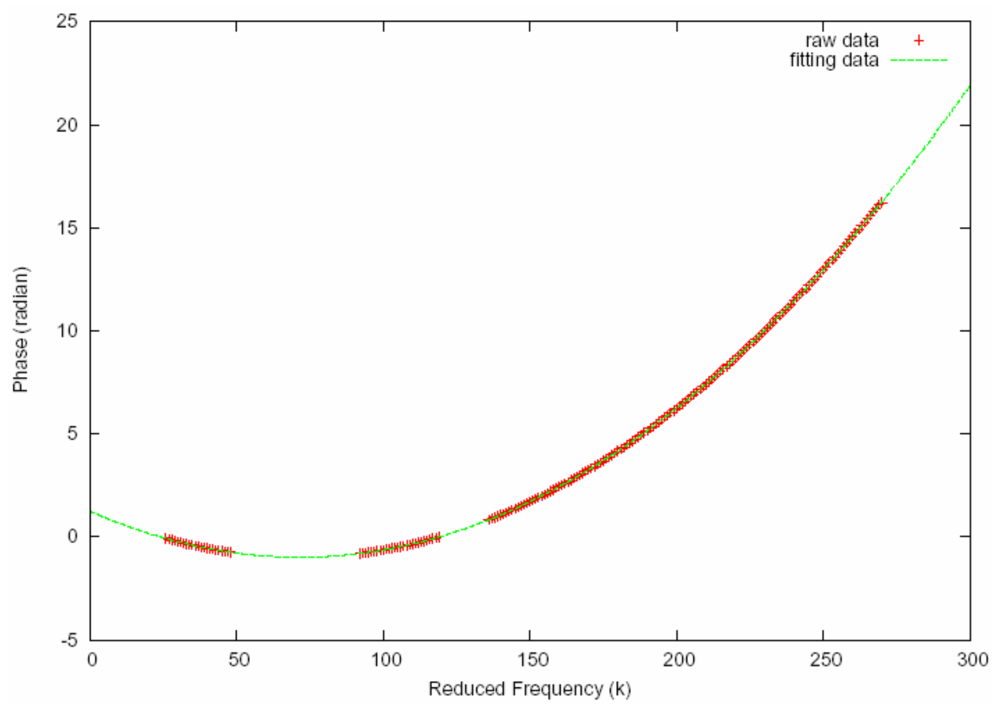

Figure 7. Result of the phase fitting procedure. The red crosses represent the raw phase data and the green line is the fit phase. The fitting parameters are $C_0 = 1.257$, $C_1 = -6.000$ and $C_2 = 4.000$.

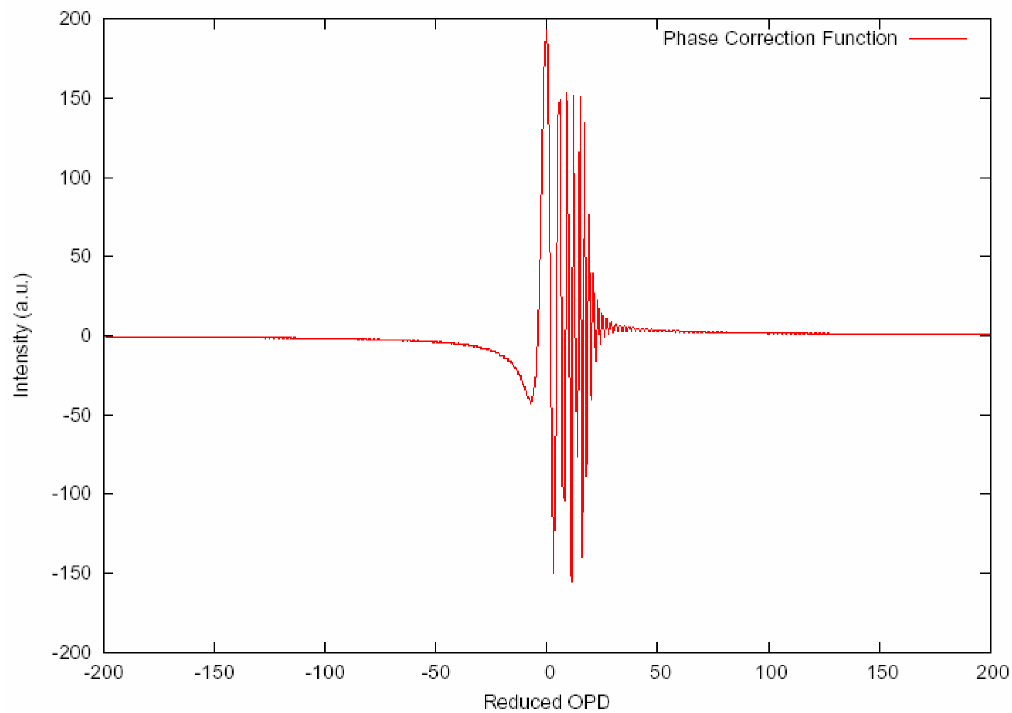

Figure 8. The Phase Correction Function with $C_0 = 1.257$, $C_1 = -6.000$ and $C_2 = 4.000$.

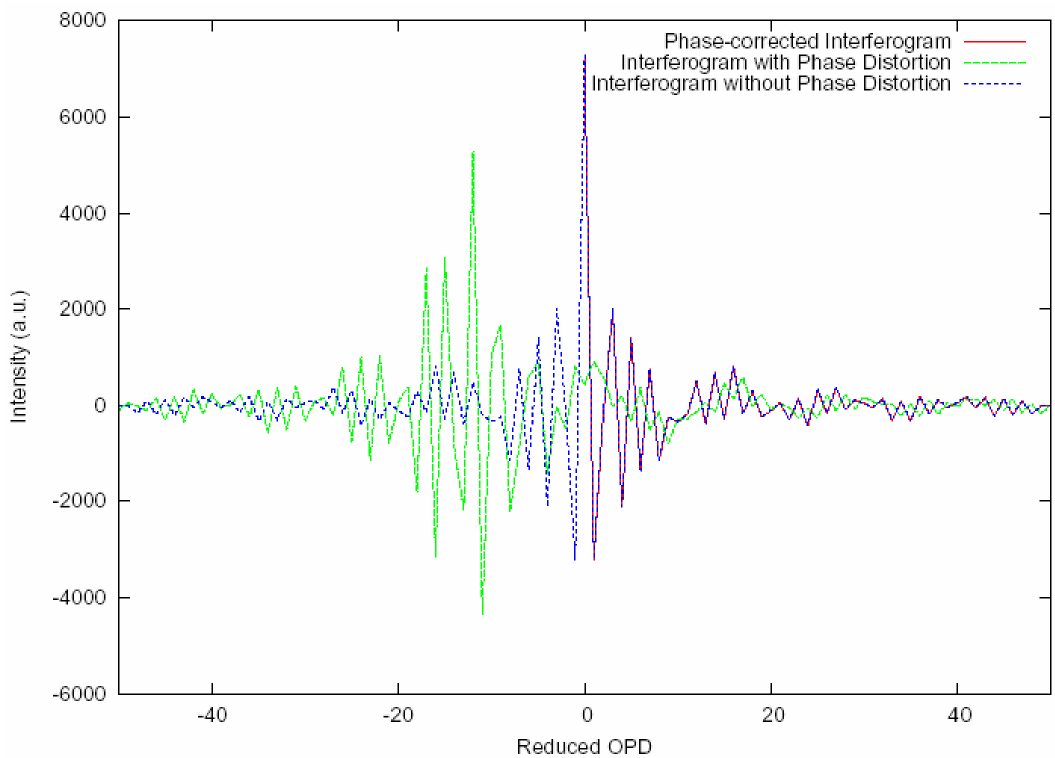

Figure 9. Interferograms around the ZPD location. The red line represents the phase-corrected interferogram (no value in the left side of the ZPD), the green dash line represents the interferogram with phase errors (C_0 =0.4π, C_1=-6.0, C_2=4.0), and the blue dot line represents the interferogram without phase errors (C_0=0, C_1=0, C_2=0). In the right side of the ZPD, the red line and the blue line overlays each other.